Does Attrition Behavior Help Explain the Duration of Interstate Wars?

A Game Theoretic and Empirical Analysis

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Abstract

Does attrition behavior, defined as waiting for the other side to give in despite the costs of delay, help explain the duration of interstate wars? To answer this question we develop a war and bargaining model that integrates a dynamic progression of the war and allows the rivals to make offers at a time of their choosing while they fight. The model predicts that, in equilibrium, states choose to fight without making significant offers in the hope that the other side will give in to outstanding demands. The model also predicts that each side's flow cost of war increases with the probability that the other side gives in to its demands. This is the testable consequence of attrition behavior. Our statistical analysis suggests that both challenger and defender include attrition behavior in their conduct of warfare, and that this behavioral element significantly affects the duration of interstate wars.

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If states could foresee the outcome of war, they should agree on a bargain without fighting and would be better off for it. Efforts to explain war between fully informed parties notwithstanding,[1] the common sense logic of this proposition puts uncertainty about the other side's priorities or resolve at the center of modern explanations of the decision to fight. The modern bargaining and war model brings bargaining into the overall picture using it as the instrument that prompts the other side to reveal its true type. The duration of war is then the time it will take to conclude the screening of the adversary for type through the observation of battle outcomes and the subsequent offers of negotiation (see Filson and Werner 2002, Slantchev 2003b and Powell 2004).[2] The bargaining model of war requires that rivals bargain while they fight, and builds on the relationship between battle outcomes and bargaining positions as emphasized by Wagner (2000, 2007). But the screening process developed in the war and bargaining model is at odds with what we know about states at war.

Pillar's (1983) detailed examination of bargaining between states at war finds that the parties negotiated *before* an armistice in only nineteen of the one hundred and forty two interstate wars examined. And even when states engaged in pre-armistice negotiations "it was only after a period of fighting without talking that the parties began to talk.." (Pillar 1983, 44). For example, the Indochina war had lasted for eight years before France and the Viet Minh entertained a "first open and direct contact" over the exchange of prisoners in July 1954.[3] At the close of the Russo-Japanese war of 1904, Japan and Russia finally agreed to talk at Portsmouth in July 1905 having fought without talking for over a year. As White (1964, 206) reports, by early 1905 "the willingness of the Russian government to seek peace was as half-hearted as its desire to avoid war had appeared." Clearly, states fight *without* negotiating for much of the duration of a conflict. As Fearon puts it, there is a "nearly universal tendency for states at war.....to simply fight for extended periods of time without making serious offers for a negotiated settlement," (Fearon 2007, 26). But, then, if bargaining plays little role in the conduct of actual wars how should we explain their duration?

Fearon (2007) and Smith and Stam (2004) offer two different explanations for states' failure to make serious settlement offers while they fight. Fearon (2007) describes a "ratchet effect" to explain the failure of bargaining in civil conflicts: Because governments cannot commit to honor a negotiated settlement that only the weak would accept, battle becomes the privileged instrument of screening. As a result, governments will fight rebel groups without even offering settlements that would be acceptable to the weakest rebel group, counting instead on the outcome of battle to sort out the weak from the strong. Smith and Stam (2004) do not appeal to commitment problems but, instead, describe states who entertain divergent beliefs about their ability to win battles. As long as the protagonists disagree on their relative strength, with State A believing that it is stronger than State B believes that it is, B's settlement offers will be unacceptable to A.

We propose a third explanation: states at war may simply delay negotiations in the hope that the other side will give in to outstanding demands *instead* of attempting to bargain, a strategy that we refer to as attrition behavior. By deciding to fight without negotiating, each party tests the other's resilience by inflicting costs without offering a way out through compromise. Attrition behavior refers to the *strategic* "act of weakening through constant attack" that may lead the other side to back down before you do.[4] Malkasian (2004), in his analysis of military strategy generally and the Vietnam and Korean wars in particular, claims that attrition warfare, implemented in an attempt to compel the other side to accept one's bargaining position, should be "assessed as a basic process in warfare" (Malkasian 2004, 942). If attrition behavior is a "basic process" in the conduct of war, it should affect the length of wars, and war and bargaining models should account for it.[5]

The proposition that warring states forgo bargaining in order to wait it out, hoping that the other side will give in, requires theoretical and empirical support. We provide this support in two steps: First, we extend the classic war of attrition model to allow for bargaining and for the possibility of military victory. We show that, in equilibrium, the warring parties *choose* attrition behavior instead of looking for a bargain that both parties can accept to end the war.[6] Second, we scan the data on the duration of interstate wars for traces of attrition behavior as predicted by the model.

Attrition behavior involves weighing the expected costs against the chances that the other side will give in to one's demands. The lower the *believed* likelihood that the opponent will give in, the lower the flow costs one should be willing to bear for the same prize. If beliefs are an accurate prediction of rational behavior, this implies that higher war flow costs suffered by one side are associated to a higher probability of surrender or concession by the other. This is not a statement about battle deaths or military capabilities as they affect the course of the war. Rather it is a statement about a state's willingness to commit resources to the war effort given the other side's perceived willingness to fight on.

Testing for attrition behavior requires that we use a comprehensive measure of a state's investment in war as a predictor of the other side's willingness to give in. One such measure is the proportion of a state's population engaged in military activities. Not only are these individuals at risk for their lives but they are unable to contribute to their country's economic growth while they fight. Our war and bargaining model suggests that a state will be more likely to give in to enemy demands as the enemy's military as a proportion of its population grows. Of course this is not a statement about the relative size of enemy forces as large states can fight smaller ones. Thus a state with a high proportion of the population in the military may have a small army.[7] Our analysis of 72 interstate wars that have occurred between 1823 and 1990[8] reveals that once traditional drivers of duration such as battle deaths, capabilities and terrain are accounted for, the proportion of a state's population in the military positively, and significantly, affects the other side's willingness to give in. We therefore conclude that attrition behavior helps to explain the duration of interstate wars.

I. The Model

1. Structure and Assumptions

The classic two player war of attrition model (Fudenberg and Tirole 2000,119-26) captures the following interaction: the players engage in a costly competitive process to capture a prize. The player who waits long enough for the other to drop out of the race wins the prize. The strategic decision to be made is therefore one of timing: when should I quit given my opponent's expected resilience. This game of timing is usually set in the time-continuum to avoid the behavioral constraints implicit in discrete-time turn-based models. The war of attrition model has been applied in a number of contexts. Fudenberg *et al* (1983) applied it to patent races and more recently Smith (1996) applied it to sanctioning behavior. To better capture various aspects of interstate warfare, we develop the classic war of attrition model in two important ways: battlefield outcomes evolve with time, and, as the players fight, they can negotiate freely adjusting offers and counteroffers with battlefield gains or losses.

1.1 War Dynamics

A war and bargaining model should not ignore the possibility that the contest be settled on the battlefield before any compromise is reached. This is important if our prediction of attrition behavior is to be valid in all circumstances. In keeping with the time-continuum framework, we describe the war dynamics using the differential equation approach pioneered by Lanchester (1916): Let $z_i(t) \geq 0$ represent side i's war effort which we think of as the proportion of i's population engaged in the military at time t. Side i has a maximum size $b_i > 0$ that will be assumed invariant during the war episode. The difference $(b_i - z_i)$ is what can be mobilized at rate a_i. In the absence of combat losses, z_i would grow at rate $z_i' = a_i(b_i - z_i)$ from an initial size $z_i(0) = z_i^0$. But once combat has started, the opponent j's forces can destroy, disable or capture i's forces with a kill rate $k_j > 0$, proportionally to j's war effort z_j. The resulting evolution equation for side i reads (as long as z_i and $z_j > 0$)

$$z_i' = a_i(b_i - z_i) - k_j z_j \tag{1}$$

If side i is defeated completely by side j at time t, $z_i(s) = 0$ for all $s \geq t$.[9] A symmetric equation holds for side j. This system of differential equations is formally identical to the so-called generalized Lanchester equations (Bellany 1999).

The evolution of the system depends on the quantities $q_i = (a_i b_i - k_j b_j)$ that measure the difference between the maximum growth rate $a_i b_i$ of i's forces and their maximum kill rate $k_j b_j$ by j's forces. There are three essential cases:[10]

(a) if $q_i > 0$ and $q_j > 0$ then the system evolves towards a stalemate (z_i^*, z_j^*), with $z_i^* = \frac{a_j(a_i b_i - k_j b_j)}{a_i a_j - k_i k_j}$ from any starting values;

(b) if $q_i > 0$ and $q_j < 0$ (conversely $q_i < 0$ and $q_j > 0$) then the system evolves toward a complete annihilation of j's forces (conversely i's forces) starting anywhere;

(c) if $q_i < 0$ and $q_j < 0$ then the system evolves generically to the complete annihilation of one side or the other depending on the initial force ratio.[11]

1.2 Costs and Utilities

The costs of war can be thought of in terms of the opportunities lost by fighting. Mobilized men and women are no longer available to engage in economic growth enhancing productive activities, and are engaged instead in activities that can lead to the destruction of property, the displacement of civilian populations and the death and maiming of the troops. These flow costs $c_i(z_i, z_j)$ for i clearly increase as i commits a larger proportion of its resources to military activities. Thus $c_i(z_i, z_j)$ increases with z_i and remains strictly positive as long as the players fight. But it reduces to zero when fighting ceases.[12]

Utility $u_i(x_i)$ is assumed increasing with share x_i.[13] We assume that x_i remains constant until the conflict is over and a new share is agreed upon so that share is a step function $x_i(s)$. As is standard in the scholarship on war, we also assume that war costs c_i outweigh the benefits of fighting so that, as long as war lasts, player i receives a negative payoff flow

$$v_i(s) = u_i(x_i(s)) - c_i(z_i(s), z_j(s)) < 0 \qquad (2)$$

Each player's objective is a discounted sum of present and future payoff flows with discount factor e^{-r_i} ($r_i > 0$)

$$\pi_i = \int_0^\infty r_i e^{-r_i s} v_i(s) ds \qquad (3)$$

It will be useful to rewrite formula (3)

$$\pi_i = \sigma_i(t) + e^{-r_i t} \pi_i^t \qquad (4)$$

The two terms in (4) represent a sunk cost $\sigma_i(t) = \int_0^t r_i e^{-r_i s} v_i(s) ds$, and an expected payoff $\pi_i^t = \int_0^\infty r_i e^{-r_i s} v_i^t(s) ds$ with $v_i^t(s) = v_i(t+s)$. So, a player's discounted objective is the sum of a sunk cost $\sigma_i(t)$ of waiting until date t and a discounted objective π_i^t

starting at time t that is *structurally* identical to π_i.[14] If some permanent settlement is reached where i receives the constant share x_i^* then $\pi_i^t = u_i(x_i^*)$ since it has constant payoff flow $u_i(x_i^*)$. The settlement can result from an accepted offer or from the complete military defeat of one side. If, for instance, $z_j(t) = 0$ (while $z_i(t) > 0$) then j is completely defeated and i enjoys $\pi_i^t = u_i(1)$ since $x_i^* = 1$.

1.3 Bargaining and Fighting in Continuous Time

An offer by j to player i must specify the magnitude y_j that i would enjoy permanently if he accepted it. In order to respect the common sense principle that an offer must be made *before* it is accepted we assume that it must remain on the table for some time after it is made.[15] But it is entirely up to j to decide how long the offer lasts since j is free to withdraw or change it at any future time. Offers are therefore functions of time and we denote by $y_j(t)$ what j offers i at time t. Technically, we allow y_j to be left-discontinuous if j decides to make a quantum change in his offer, such as withdrawing it entirely. But we require that y_j remains constant or evolves at most *smoothly* within intervals $[t, t+s)$ chosen by j. A smooth evolution can result from the phrasing of an offer, such as accepting (smoothly evolving) battlefield positions.

To be rationally acceptable, an offer must be at least as good as the other side's Best Alternative to a Negotiated Settlement, BATNA. Formally we define player i's BATNA $\xi_i(t)$ at time t by

$$\xi_i(t) = u_i^{-1}(max\{0, \pi_i^t(W)\}) \quad (5)$$

where $\pi_i^t(W)$ is the expected payoff of fighting to the finish.

In most cases player i's BATNA reduces to $\xi_i(t) = 0$ which amounts to giving up the entire prize and incurring no further costs of fighting. However, if the dynamics of war are leading one player to victory, the nearing prospect of winning the whole prize could outweigh the remaining costs of war for the victor. The victor's BATNA would then be strictly positive. Offering nothing beyond the other side's BATNA ($y_j \equiv \xi_i$) is what we interpret as making no concession.

As the players make new decisions on offers, acceptance and the pursuit of war, the game transits from one state to another moving along a path which can be described as a succession of smoothly evolving states separated by discrete events. An event is a player-induced switch from one state to another and typically involves a change of payoff flow. It will be useful to characterize paths with reference to the player who moves the game from one state to the next. For instance, a wartime acceptance by j of an offer by i of share y_j moves the game to a state of peace and yields the constant payoff flow $u_i(1-y_j)$ to player i. Path ω_j^s may then mean an ongoing state of war, with a standing offer by i to j of share y_j, followed by j's acceptance of that offer at time s.

In the continuous time framework, the players' strategies involve not just the choices they make, such as the magnitude of an offer or a declaration of war, but the exact timing of these moves. j's timing s of the move to ω_j can always be described by a distribution function $\phi_j : [0,\infty) \to [0,1]$. If the timing is deterministic, ϕ_j jumps from 0 to 1 at precisely time s. But timing can be probabilistic either if mixed strategies are allowed or if incomplete information translates into beliefs about which type $j = \phi_j(s) \in [0,1]$ makes the deterministic move to ω_j at time s. At each point in time, player i can wait for j to move to ω_j or can himself decide to move to another state ω_i at some time t. Typically, ω_j is more desirable to i than ω_i but the value of moving to ω_i, typically i's acceptance of j's outstanding offer, also erodes with time. If $\pi_i(\omega_j^s)$ denotes the expected payoff of path ω_j^s as defined in (4) i's problem is to optimize over time t

$$\mathbb{E}\pi_i(t) = \int_0^t \pi_i(\omega_j^s) d\phi_j(s) + (1-\phi_j(t))\pi_i(\omega_i^t) \qquad (6)$$

The integral in (6) is the expected value that the move to ω_j occurs before i moves to ω_i while the second term is the expected value of the converse. Note that if victory by either side is expected at time θ, $\mathbb{E}\pi_i(t) \equiv \mathbb{E}\pi_i(\theta)$ for all $t \geq \theta$ and the earliest optimal t never exceeds θ.

2. War and Bargaining in Equilibrium

2.1 Rational Behavior

A rational player i should optimize $\mathbb{E}\pi_i(t)$ by his choices of timing t for the move ω_i^t and of his offer function y_i, given his beliefs and expectations of the other side's behavior, together with the expected costs and outcome of war. We also require that rationality holds for all *possible* developments of the game, not just those that would result from the *planned* developments.

The complete information framework is easiest to discuss and uses the concept of subgame perfect equilibrium (SPE). But it raises some conceptual issues. If only pure strategies are allowed the game only has trivial solutions: one side or the other gives up entirely ($\phi_j(t) = 1$) and the sooner ($t = 0$) the better. This would never lead to war and would fail to account for the empirical evidence. But if mixed strategies are allowed they need to be given conceptual justification. In essence, a mixed strategy is a statement that a player will make choices with probabilities. But all too often, one side's probabilities in equilibrium seem determined by the need for the *other* side to be indifferent between its own randomized choices. This has led to the criticism that a player's rational behavior should be driven by his need to maximize his own payoff, not by the desire to widen the scope of his opponent's optimal choices by making him indifferent.

Another interpretation, proposed by Harsanyi (1973), is that the probabilities in one side's mixings are really the other side's beliefs about the type of opponent he is facing in a game with slight uncertainty about the stakes. Each opponent's *type* then uses a *pure* strategy that depends on its own payoff parameters. But this requires an incomplete information model and the concept of perfect Bayesian equilibrium (PBE).

This process of purification of an SPE is highly technical. So we discuss our findings conceptually in the text and provide justifications in the appendix.

2.2 Bargaining

Acceptance of the other side's terms can occur at any time during the game. If it involves non-trivial distributions, it cannot be expected to occur with certainty at any given date. In any mixed strategy SPE, each player optimally accepts the other's current offer with a probability that depends on the distance between their respective bargaining positions as well as the costs incurred by the other side. Each player waits despite the cost of fighting, in the hope that the other side will give in first. This is attrition behavior. In general, side j's hazard rate is given by

$$\frac{D_+\phi_j(s)}{1-\phi_j(s)} = \lambda_i(s) = \frac{r_i\big(u_i(y_j(s))-v_i(s)\big)-D_+u_i(y_j(s))}{u_i(1-y_i(s))-u_i(y_j(s))} \tag{7a}$$

where D_+f denotes the right-hand side derivative of f with respect to s.[16] $\lambda_i(s)$ therefore describes j's instantaneous probability of accepting $y_i(s)$ or the hazard rate associated to j's acceptance of i's terms. In such an SPE, the parties can adopt a wide range of bargaining positions and make plans to offer compromises.

The situation changes substantially as uncertainty is introduced. $\phi_j(s)$ is no longer a part of j's strategy. Instead, it expresses i's beliefs about what type of j he is facing. i's uncertainty about j can take various forms. We develop in appendix the case where the uncertainty is about the players' flow costs but a more general picture involving utilities and discounting is possible as well. Because beliefs about j affect each type i's optimal behavior, which in turn affects j's beliefs about i, the beliefs are really entangled and (7a) ends up being replaced by

$$\frac{D_+\phi_j(s)}{1-\phi_j(s)} = \lambda_{i=\phi_i(s)}(s) \tag{7b}$$

with the same functional form for λ_i. Evidently, such beliefs converge to the mixed strategy expressed by (7a) as the magnitude of uncertainty approaches zero since all types i will reduce to a single player.

What changes with the introduction of uncertainty is primarily the nature of the players offers. We find that if one side's offer is continuous in time then the other's can only be continuous and will lack generosity: A player who is in a stalemate or is headed for imminent victory will offer the other side no concession. And a player who is headed for defeat will do the same until a point in time when his defeat is so imminent that he will offer only a small concession above what the victor can already guarantee himself. As uncertainty vanishes that behavior persists and the victor will not even enjoy the slightest concession above his BATNA.

This is of interest in light of Pillar's (1983) findings and of Fearon's observation that States tend to "simply fight for extended periods of time without making serious offers for a negotiated settlement" (Fearon 2007, 26). This is also in line with the evidence on wars of insurgency that can easily turn into stalemates and suggests an alternative to their prevailing explanation as a commitment problem. It also agrees with the observation that the making of offers takes place during pre-armistice negotiations as fighting has revealed the other side's prospects. For example, Britain's concession of a territorial *status quo ante bellum* at Ghent following America's victory at the battle of Plattsburg in September of 1814 can be interpreted as granting America concessions that were only slightly better than the alternative of continued hostilities.[17]

II. The Empirical Evidence

1. The Data and the Relationship to be Tested

Our war of attrition model predicts that state j in a militarized dispute is more likely *to be observed* to give in to i's demands as the flow costs of war incurred by i

increase. The relationship between j's hazard rate and i's war costs is a flow statement. It is not the accumulated costs borne by i that prompt j to give in as time goes by and the war drags on. Rather, it is the costs willingly incurred by i in each war year that should be positively correlated to j's perceived probability of acquiescence to i's demands.

It is worth clarifying the underlying logic of this claim once more. We develop it for the defender. If the defender is rational, he will weigh the flow costs associated to his war effort against what he believes is the probability that he will observe a surrender of the challenger. A more intense effort on the defender's part must correspond to his belief that the types of challengers he faces will give in sooner. In perfect Bayesian equilibrium the defender's beliefs are an accurate prediction of the challenger's type and the probability that he will concede. The reasoning applies identically to establish the relationship between the challenger's flow war costs and the probability that the defender will give in. Our empirical investigation of the relationship between i's flow war costs and j's hazard rate therefore tests for attrition behavior as well as rationality.[18]

To ensure that our results can be compared with those obtained by others, we have used a subset of the data used by Slantchev (2004) to demonstrate the importance of uncertainty on war duration, selecting all bilateral militarized disputes. We exclude the multilateral cases since our theory does not cover them. The data base comprises 72 wars initiated between 1823 and 1990 and 138 war years. In order to test for attrition behavior we sort the data to distinguish between those wars that end with the challenger backing down, from those that end with the defender giving in. This sorting is necessary if we are to find trace of attrition behavior. Indeed, to identify this behavior we must test for a positive relationship between the losing state's estimated hazard rate and the *other side's* costs. This relationship could not be observed in a data base that groups all wars.

We used Slantchev's coding of war outcome to distinguish between those wars that ended with the challenger acquiescing to the defender's demands from the wars that end with the defender giving in to the challenger. Slantchev codes the outcome of war on a scale of 1 to 4. Wars that end in outcomes 1 and 2 end with the challenger giving in to the defender, having been defeated on the battlefield or having agreed to a cease fire. There are 27 such wars representing 77 war years. Wars that end in outcomes 3 and 4 end because the challenger is victorious or obtains substantial concessions from the defender. We consider that these wars end with the defender giving in to the challenger. There are 45 such wars representing 61 war years. We analyzed the duration of these two sets of wars using a proxy for the other side's war costs as an explanatory variable together with measures of own military capability, political regime and battle deaths as well as war condition measures such as terrain and contiguity.

2. A Proxy for Flow War Costs

We used militarized personnel as a share of the population for each state as an indicator of the sacrifice endured by the country as a result of war. Not only are military personnel at risk for their lives, these individuals are not able to contribute to the country's economic growth while they fight. Of course, fighting in and of itself will stimulate the military industrial complex, but in a wider view of social welfare a population's preference for butter over guns suggests using the share of the population at war as one possible indicator of war cost. This proxy for i's costs is also directly comparable to variable z_i, and our model predicts that j's hazard rate increases with i's costs c_i which are themselves increasing in z_i.

Importantly our choice of proxy for war costs is not correlated with other key explanatory variables that measure performance on the battlefield. For example, the COW capability index or size of the military are not correlated to the proportion of the population in the military for the warring states in our data set. The absolute size of the military forces together with the COW capabilities index capture fighting ability and these variables will affect the duration of warfare through their impact on battlefield outcomes. By contrast, the proportion of the population in the military captures a broader index of war costs and our hypothesis is that, if attrition behavior is to be observed, it will involve this measure of costs. It is also important to note that the proportion of the population in the military cannot be considered a proxy for issue salience as it is either weakly correlated to salience or negatively correlated to salience for each state.[19]

As a first rough test of the model's predictions, we used the non parametric Kaplan-Meier estimate of the survivor function for our data. We estimated survivor functions for those wars for which the challenger backed down (27 wars,77 war years) separating the data according to *defender* costs measured as the proportion of the *defender's* population in the military. For those wars that end with the defender acquiescing to the challenger's demands (45 wars, 61 war years) the data was separated according to the *challenger's* costs. The graphed Kaplan-Meier survivor functions are reproduced below:

<<<Graph 1 and 2 about here>>>

The estimated survivor function for those wars for which *the other side* incurred *above* average costs lie clearly below the survivor functions for those wars for which *the other side* incurred *below* average costs whether the war is ended by the challenger or the defender. In other words, the defender gave in sooner when the challenger incurred above average war costs while the challenger gave in sooner when the defender incurred above average war costs. This is what attrition behavior would predict. It is important to note that no such relationship is identified if we examine *challenger* survivor functions according to war costs incurred *by the challenger* or defender survivor estimates according to defender costs. As we mentioned above, our proxy for war costs is not highly correlated to capabilities or absolute size of the military. The same can be said for the correlation between defender and challenger proxies for war costs. In sum, our proxy for i's war cost does not primarily reflect i's strength and is not correlated to j's war costs or strength. Instead we interpret it to capture i's willingness to incur costs to force j to give in to its demands.[20]

3. Control Variables

Following Bennett and Stam (1996) we included descriptors of military might as well as political and geographical control variables to explain the duration of those wars that end with target acquiescence to the initiator's demands:

2.1 Military Might and the Experience of War

We consider three relative measures of military might and compute them for the challenger (prefacing the measure by c) and for the defender (prefacing the measure by d).

- **Relative military capability,** *crelativecap,* and *drelativecap* take each side's COW capability index and divide it by the sum the two side's capability indices. The probability that i will give in to j's demands ending the war is expected to decrease with i's relative capability.

- **Relative flow rate of loss**, *crelativeflowloss,* and is *drelativeflowloss* divides each side's ratio of target deaths to military personnel each year by the sum of this same ratio for both sides. A higher relative rate of loss for i, is expected to hasten i's acquiescence and therefore shorten the war.

- Following Bennett and Stam (1996) we measure the **relative quality of military forces**, *crelativequal* and *drelativequal*, as the ratio of one side's real military expenditures per person in the military to the other's**.** As argued by Bennett and Stam

(1996) following Huh, Bennett and Gelpi (1992) and Stam (1996), expenditures per soldier measure the quality of the state's military forces and a larger quality difference between i and j should shorten the war by increasing the probability that i will back down.

2.2 Geographic and Political Variables

• Following Bennett and Stam (1996) **terrain** is expected to have an impact on war duration. **Terrain** difficulty has been coded by Slantchev(2004) for the data base we use here following the methodology in Stam (1999). Higher values for the variable *terrain* correspond to more difficult geographic conditions. Wars fought on difficult terrain are expected to last longer.

• The COW **contiguity** score is used to measure the difficulties that might be involved in supplying the troupes. Variable *contiguity* ranges from a 1 if the states at war are neighbors on land to 6 if the warring parties are separated by vast oceans. A low value for *contiguity* is expected to favor the parties' willingness to continue the fight and should be associated to longer wars. However, to the extend that the battle takes place on the defender's territory, one might anticipate that the challenger's supply difficulties would harden the defender's resolve leading him to be less likely to give in. We therefore anticipate that the impact of *contiguity* will differ depending on whether the war ended with the challenger or the defender giving in.[21]

• We use **issue salience**, *csalient,* and *dsalient* as coded by Slantchev (2004) following Holsti (1991). This variable is coded 2 if the issue is deemed critical, 1 if it is of medium salience and 0 if it was not salient.[22] We expect a higher issue salience for i harden i's resolve and delay its acquiescence to the j's demands.

• The political regimes of both challenger and defender are expected to have an impact on the each state's willingness to give in. Bennett and Stam (1998) and Reiter and

Stam (2002) argue that democratic challengers are less likely to continue fighting as the war drags on because the political costs of mounting casualties are higher for democracies than for autocracies. To capture this we construct variables *caudience* and *daudience* that multiply each state's casualties by its democracy score (Marshall and Jaggers 2005). It is expected that the probability that the challenger gives in increases with *caudience* while the defender will give in with higher probability as *daudience* increases.

4. Results

4.1 The Statistical Setup

We estimated duration models assuming a Weibull distribution for the unknown base hazard.[23] Our models are estimated using one observation per war year. Indeed, our measures of cost to the challenger and defender vary over the years in which a given war is fought. The use of a time varying covariate data set therefore increases the information input while also increasing available data points. We present two sets of estimations, one for the wars that end with the challenger backing down and one for those wars that end with the defender backing down. In both cases we find significant evidence of attrition behavior in the duration models that we estimate although the two sets of data have very different characteristics. The wars that end with the challenger giving in are typically much longer than those that end with the defender giving in. Indeed the mean duration of the wars that end with the challenger giving in is 20.5 months By contrast the wars that end with the defender giving in last on average 3.7 months only. The distributions of war duration are skewed to the left in both cases and the data is very dispersed with standard deviations of duration exceeding mean lengths.

The two sets of data also differ when it comes to the characteristics of the explanatory variables. The relative capabilities of the defenders who give in tend to be lower than those of the challengers who give in, but they tend to suffer lower audience costs (measured by *daudience*) than the challengers. Defenders and challengers who give in also differ on their relative real dollar expenditures per soldier. That statistic is much higher on average for the defenders who give in than it is for the challengers. It is also the case that issue salience tends to be higher for the defenders that give in than for the challengers. Table 1 below summarizes some of these differences highlighting the relevant explanatory variables for each case:

<<<Table 1 about here>>>

. .We now turn to our hazard rate estimations.

4.2 The Challenger Gives In

We estimate four models. Model 1 uses the proportion of the *defender's* population in the military, together with the control variables that are relevant to the *challenger*. Model 2 adds to Model 1 by adding the proportion of the *challenger's* population in the military to the list of explanatory variables. This is to verify the relative significance of defender and challenger war costs in explaining the probability that the challenger will give in, ending the war. The proportion of the defender's population in the military can vary over the course of the war suggesting that the change in *dmilpopshare* may also be instrumental in explaining the duration of wars. To test the relationship we estimated Model 3 which adds to Model 1 the explanatory variable *dmilpopsharedpct* which measures the percentage change of *dmilpopshare*. Model 4 presents the best model from a purely statistical viewpoint and is constructed by paring down Model 1 to eliminate those variables that do not significantly add explanatory power according to a log likelihood ratio test. The results are presented in Table 2:

<<<<Table 2 about here>>>>

In all our models, the coefficient on *dmilpopshare* is positive and significant at the 99% confidence level. Our empirical estimates therefore support the hypothesis that the challenger will be more likely to give in as the *defender*'s costs increase so that increasing defender costs are associated to shorter wars. We therefore find evidence for attrition behavior in our statistical analysis of the data. Again, as discussed earlier, *dmilpopshare* is only weakly correlated with the initiator's military capabilities or the size of its army. It is also the case that *dmilpopshare* is not correlated to the relative quality of the *challenger*'s military force as measured by the ratio of the challenger's real military expenditures per soldier to the target's real military expenditures per soldier. Indeed the correlation coefficients between *dmilpopshare,* and the relative quality of the challenger's military force *crelativequal* stands at 0.1076. The same can be said of the correlations between *dmilpopshare* and all the other explanatory variables used to explain the duration of the interstate wars that ended with the challenger giving in to the defender. Increases in *dmilpopshare* over the course of a war are also positively correlated to the likelihood that the challenger will give in as shown in Model 3 but the result is weak as variable *dmilpopsharedpct* is not significant.[24]

The signs on all explanatory variables of Model 1, have the predicted signs although they are not always significant: The challenger is less likely to give in to the defender as his relative capabilities (*crelativecap)* , and the relative quality of his army increase (*crelativequal)*.[25] As predicted by Bennett and Stam (1996) the difficulty of the *terrain* is associated to longer wars as the challenger's hazard rate decreases with terrain difficulty. The challenger is also less likely to give in as the issues involved are more *salient* to the challenger as expected. Our challengers are more likely to give in when the *contiguity* index increases as the argument about the difficulty involved in supplying the troops would suggest. The coefficients on *crelflowloss* and *caudience* have the expected signs but are not significant.

Model 2 adds to Model 1 by introducing *cmilpopshare* as an explanatory variable. By doing so we are able to verify that our test of attrition behavior is not affected by introducing the challenger's war cost measure. Not only are *dmilpopshare* and *cmilpopshare* uncorrelated, but the challenger's own war costs do not impact the probability that the challenger will give in to the defender. By contrast we will see that the challenger's costs *do* impact the probability that the *defender* gives in to the challenger's demands. Model 4 pares down Model 1 to obtain the best statistical fit. While this is an a-theoretical approach to the analysis of the data, it allows us to verify that variable *cmilpopshare*, whose significance is critical to the testing of attrition behavior, remains statistically significant as explanatory variables that do not add to the power of the regression to explain duration are eliminated. Model 4 eliminates the variables of Model 1 according to significance using a likelihood ratio test. *cmilpopshare* remains significant and with the correct sign.

4.3 The Defender Gives In

Again, we estimate four models. Model 1 uses the proportion of the *challenger's* population in the military, together with the control variables that are relevant to the *defender*. Model 2 adds to model 1 by adding the proportion of the *defender's* population in the military to the list of explanatory variables and Model 3 adds to Model 1 the explanatory variable *cmilpopsharedpct* which measures the percentage change of *cmilpopshare*. Model 4 presents the best model from a purely statistical viewpoint The results are presented in Table 3:

<<<<Table 3 about here>>>

When the defender gives in, we also find evidence of attrition behavior. Indeed the proportion of the *challenger*'s population in the military is positively and significantly related to the probability that the defender will give in to the challenger's demands. All other explanatory variables except the relative quality of the defender's armed forces as measured by real dollars spent per soldier (*drelativequal*) are of the expected sign although some are not significant. Adding the proportion of the *defender*'s population in the military or the change in *cmilpopshare* as explanatory variables, or paring down Model 1 does not jeopardize the sign and significance of *cmilpopshare* as a significant factor in the explanation of the duration of wars that end with the defender giving in.

In brief, we find evidence of attrition behavior whether we analyze those wars that end with Challenger backdown or those that end with Defender acquiescence. We now assess its relative importance in explaining the duration of interstate wars.

4.4 Goodness of Fit and Model Predictions

4.4.1 Goodness of Fit

One basic measure of goodness of fit for duration models is the log likelihood value or the Aikaike Information Criterion when comparing non-nested models. Bennett and Stam (1996) also examine absolute errors and the ratio of the absolute error to the duration data when judging their models' goodness of fit to the data. As we used a subset of the data used by Slantchev (2003) we were also able to compare the goodness of fit of his model and ours directly. Table 4 provides comparative data on predicted duration as it compares to observed duration in our model and the model used by Slantchev (2003).[26] All data are in months.

<<<Table 4 about here>>>

Bennett and Stam (1996) first discussed including the mean absolute error to observed duration in an analysis of a duration model's goodness of fit. Intuitively, a smaller value for this statistic is preferred since it signals a smaller prediction error relative to the actual observed values especially if the lower mean value is associated to a lower standard deviation. Moreover, because it is normalized, this statistic can be compared across models using different data. For Bennett and Stam's (1996) best model the mean absolute error to observed value ratio is 4.42. For Slantchev's model, estimated on the subset of his data used here, the mean of the absolute error to the observed duration is 8.04 for those wars that end with the challenger backing down and 2.58 for those that end with the defender giving in. The mean absolute error to observed duration ratio is substantially lower for all the models that we estimate. Of course, Slantchev's model was designed to fit a wider set of data and our intent is not to pass judgment on the model itself. Simply, in those cases where our theoretical model can serve as a guide for the empirical analysis, our specification yields a better fit than Slantchev's model.[27]

4.4.2 How Important is Attrition Behavior in Explaining War Duration?

The impact of individual variables on war duration is non linear and depends on the values taken by the other explanatory variables in the model. As a result, each independent variable's impact cannot be assessed by simply examining the coefficient estimates of the hazard model. We examined the model's predicted war duration as each independent variable varied from its median value, holding all other independent variables at their median. We used Model 1 for both data sets. The cost variables *dmilpopshare* for the challenger and *cmilpopshare* for the defender are among the three most important explanatory variables for each data set. The table below summarizes their impact in explaining war duration:

<<<<Table 5 about here>>>

The median duration of the wars that end with the *challenger* giving in is 14.91 months. As the defender increases its population's commitment to the army from .279%

to 1.795%, war duration can be expected to decrease by 11.71 months or 78.5% of the duration of the median war that ends with the challenger giving in and 6.2% of the longest 98.7 month war. Only the challenger's relative capabilities has a larger impact and *terrain* which is recognized as an important driver of war duration has a smaller impact for this set of wars. It is difficult to make predictive inferences on troop size for a single war from this data as a given proportion of a State's population can represent vastly different troop levels. To get an idea of the troop deployments that could be involved, consider the Greco-Turkish war of 1920. Turkey, the defender, deployed 162,000 troops representing 1.3% of her population. This is close to the median of the data for *dmilpopshare*. The war lasted for 28 months. Extrapolating from our data analysis, a surge in troops to 1.79% of the Turkish population (or the 3rd percentile of *dmilpopshare*) would have sent 61,000 more men to the front and would have reduced the duration of the war by about 3 months or 11%.

The wars that end with the *defender* giving in, have a median length of 1.97 months. Explanatory variable *cmilpopshare* has an impact on duration for these wars that is comparable to that of relative capabilities. As *cmilpopshare* increases from 0.498% to 2.466%, war duration decreases by 1.06 months or 53.8% of the median and 6.7% of the longest 15.9 month war in this subset of the data

Our analysis of the predictive power of the variables that explain war duration therefore confirms the importance of attrition behavior whether we consider those wars that end with challenger or defender backdown.

Conclusion

Does attrition behavior, defined as waiting for the other side to give in despite the costs of fighting, help explain the duration of interstate wars? To answer this question we have developed a bargaining and war model that allows the rivals to *choose* attrition behavior instead of bargaining. Our model is set in continuous time and includes war dynamics that can settle the dispute on the battlefield or lead to a stalemate, as well as incomplete information about the players' stakes. We show that attrition behavior arises in equilibrium regardless of the magnitude of uncertainty. Moreover, even the slightest uncertainty incites the two sides to harden their bargaining stance. Neither side offers the other anything until one side's victory is clearly in sight. And the loser offers barely more than what the victor can guarantee himself when nearing victory. The model highlights that a pattern of fighting without talking is compatible with a bargaining approach to understanding war. Our analysis therefore offers a bridge between the observed bargaining behavior of warring states and the war and bargaining model.

Attrition behavior involves weighing expected costs of fighting against the chances of prevailing on the battlefield or at the negotiation table. The lower the perceived likelyhood that the opponent will give in, or will be defeated, the lower the flow costs of war one should be willing to bear. If *beliefs* about the opponent's resilience are an accurate prediction of his rational behavior, a proposition that must hold in equilibrium, higher flow costs incurred by one side should be positively correlated to a higher probability of surrender or concession by the other. This is the testable consequence of attrition behavior. Our statistical analysis confirms this relationship, suggesting that both challenger and defender include attrition behavior in their conduct of warfare, and that this behavior significantly affects the duration of interstate wars.

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Appendix

We first recall our main assumptions:

1) An offer $y_j \geq 1 - y_i$ is deemed instantly accepted;[28]

2) Offers remain open for some time (i.e., $y_j(t)$ has a piecewise continuous right-derivative);

3) The flow costs of war always outweigh the flow benefits ($v_i < 0$).

In the incomplete information case, we assume that each side's initial set of types is the segment $[0, 1]$. Initial beliefs are assumed uniform to simplify. Because we focus on purification, each type uses a pure strategy and we focus on (completely) separating equilibria. Since players do not forget previously held information, beliefs about side j can be described by a distribution function $\phi_j : [0, \infty) \to [0, 1]$ that gives the set of types $[0, \phi_j(t))$ that have been ruled out in the screening process by time t. Bayesian updating therefore yields uniform beliefs on the remaining $[\phi_j(t), 1]$. Constructing $\phi_j(t)$, given the two sides' respective offers, that yields optimal response times consistent with the beliefs is a first critical step. Optimizing offers is the second critical step.

The dependency of the game parameters on type is a matter of modeling choice but it should be sufficiently realistic. The only requirement for all our results to hold is that the resulting function $\lambda_i(s)$ defined in (7a) be strictly monotonous in i. A realistic case is to make all parameters independent of i, except for the costs of war given by

$$c_i(s) = (1 - \epsilon i)c_{\mathcal{I}}(s) \qquad \text{(A1)}$$

with $\epsilon > 0$ thought of as the "magnitude of uncertainty." The resulting $\lambda_i(s)$ is clearly continuous and strictly decreasing in i.[29]

In all of what follows θ denotes victory time with the convention that $\theta = \infty$ for stalemate. ω_i^t in (6) means fighting until i accepts $y_j(t)$ or a victory is reached by either side and i receives $\xi_i(t)$ (and similarly for ω_j^s). If $\phi_j(s)$ is the distribution function (interpreted as belief or mixed strategy) for the event ω_j^s, we can denote by $\phi_j^t(s)$ the distribution of the event $\omega_j^{s|t}$ "accepting at time $(t+s)$ conditional on reaching time t", by $y_i^t(s)$ i's offer at time $(t+s)$ conditional on reaching time t and by $\pi_i^t(\omega_j^{s|t})$ i's payoff viewed from time t on. The laws of probabilities yield

$$1-\phi_j(t+s)=\big(1-\phi_j(t)\big)\big(1-\phi_j^t(s)\big) \qquad \text{(A2)}$$

This yields a first result that simplifies the discussion of sequential rationality.

Lemma 1: For all $t \geq 0$ and $s \geq 0$ one has

$$\mathbb{E}\pi_i(t+s)=\int_0^t \pi_i(\omega_j^\tau)d\phi_j(\tau)+\big(1-\phi_j(t)\big)\big(\sigma_i(t)+e^{-r_i t}\mathbb{E}\pi_i^t(s)\big) \qquad \text{(A3)}$$

where $\mathbb{E}\pi_i^t(s)=\int_0^s \pi_i^t(\omega_j^{\tau|t})d\phi_j^t(\tau)+(1-\phi_j^t(s))\pi_i^t(\omega_i^{s|t})$

noting that $\mathbb{E}\pi_i^t(s)\equiv u_i(x_i^*)$ if $t \geq \theta$.[30]

So, i's expected payoff of waiting time $(t+s)$ is the sum of an expected payoff of j accepting within $[0,t)$, an expected sunk cost $\sigma_i(t)$ and a discounted expected payoff $\mathbb{E}\pi_i^t(s)$ viewed from time t on that is *structurally* identical to $\mathbb{E}\pi_i(t)$. Sequential rationality translates into the following principle (where $0 \leq t \leq t_i$): t_i is an optimal waiting time and $y_i(t+s)$ is an optimal offer for i, both viewed from time $t=0$, only if $(t_i - t)$ is an optimal waiting time and $y_i^t(s)$ is an optimal offer for i viewed from time t. We can therefore limit our discussion to the optimization of $\mathbb{E}\pi_i(t)$ over t with the understanding that the same arguments apply to that of $\mathbb{E}\pi_i^t(s)$ over s, given t. In particular, we may arbitrarily set any current time as "time 0" to simplify notation.

As previously observed, (left) discontinuities in y_j are allowed but yield Dirac masses in D_+y_j and discontinuities in ϕ_j. We initially assume *continuous* offer and distribution functions. We construct SPEs and PBEs under this assumption and later discuss the effect of discontinuous deviations. In the incomplete information case discontinuities turn out to be suboptimal responses to continuous ones.[31] The next result is key to characterizing SPEs and PBEs through the relationship between y_i, y_j, ϕ_j and λ_i.

Lemma 2: If y_j and ϕ_j are continuous and $t < \theta$ (victory time) then

$$\mathbb{E}\pi_i(t) = u_i(y_j(0)) \quad \text{(A4)}$$
$$+ \int_0^t e^{-r_i s}(1 - \phi_j(s))(u_i(1 - y_i(s)) - u_i(y_j(s)))\left(\frac{D_+\phi_j(s)}{1-\phi_j(s)} - \lambda_i(s)\right)ds$$

with $\lambda_i(s)$ as in (7a).

Proof: Since $(1 - \phi_j(t))\pi_i(\omega_i^s)$ is thus continuous and right-differentiable, rewrite $(1 - \phi_j(t))\pi_i(\omega_i^t) = \pi_i(\omega_i^0) + \int_0^t D_+\pi_i(\omega_i^s)ds$, observing that $\pi_i(\omega_i^0) = u_i(y_j(0))$, $\phi_j(0) = 0$, expressing the derivative $D_+\pi_i(\omega_i^s)$ and factoring. □

Clearly, the integrand in (A4) has the sign of the last parentheses, since $\phi_j(s) < 1$ and $y_j(s) < 1 - y_i(s)$ as long as neither side has accepted. In the *complete* information case, this provides an obvious family of SPEs as follows.

Definition 1: Let *j*'s strategy (and symmetrically for i) be given by:

(1) Fight while offering $y_j(t) \in [\xi_i(t), 1 - y_i(t))$ such that $\lambda_i(t) \geq 0$ (a.e.);[32]

(2) Reject any $y_i(t) < \xi_j(t)$ or such that $\lambda_j(t) < 0$;[33]

(3) Otherwise accept $y_i(t)$ by time t with distribution $\phi_j(t) = 1 - e^{-\int_0^t \lambda_i(s)ds}$.

Theorem 1: The strategies of Definition 1 form a SPE.

Proof: We show that i does best by adopting the symmetric strategy. Since $\lambda_i(s) \geq 0$ the given ϕ_j satisfies (7a) in $[0, t)$ and, by Lemma 2, $\mathbb{E}\pi_i(t) \equiv u_i(y_j(0))$. Indeed, this generalizes to $\mathbb{E}\pi_i^t(s) \equiv u_i(y_j(t))$ as in Lemma 1. So, *j*'s strategy defined by y_j and ϕ_j holds i to the expected payoff $u_i(y_j(t))$ at all times t regardless of what i chooses for y_i and ϕ_i. And since $y_j(t) \geq \xi_i(t)$, i can accept at any time, in particular according to the symmetric ϕ_i. If, instead, j offers $y_j(t) < \xi_i(t)$ then i is strictly better off fighting to the end and optimally rejects. And if $\lambda_i(t) < 0$ then the integrand in (A4) is strictly positive at t and i is better off waiting. Clearly, offering $y_i(t) > 1 - y_j(t)$ is counterproductive since it is instantly accepted by j and yields less than $u_i(y_j(t))$. Finally, if i offers $y_i(s)$ such that $y_i(s) < \xi_j(s)$ or $\lambda_j(s) < 0$ (a.e.) in some interval $[0, t)$ then j never accepts and $\phi_j \equiv 0$. There are only three cases at t: (1) one side is victorious and $\mathbb{E}\pi_i^t(s) = u_i(\xi_i(t))$; (2) i returns to the behavior of Definition 1 and $\mathbb{E}\pi_i^t(s) = u_i(y_j(t))$; and (3) t is infinite. In all three cases (A3) written for t reduces to

$$\mathbb{E}\pi_i(t+s) = \sigma_i(t) + e^{-r_i t}\mathbb{E}\pi_i^t(s) = \pi_i(\omega_i^t) \tag{A5}$$

Now, for any other $\phi_j \geq 0$ within $[0, t)$ we have by (A3) and (A5)

$$\mathbb{E}\pi_i(t+s) = \int_0^t \pi_i(\omega_j^\tau)d\phi_j(\tau) + \big(1 - \phi_j(t)\big)\pi_i(\omega_i^t) \tag{A6}$$

One easily verifies that $\mathbb{E}\pi_i$ is always greater in (A6) than in (A5) as long as $\pi_i(\omega_i^t)$ is non-increasing in t, which results from j's assumed choice for $\lambda_i(t) \geq 0$.[34] It is therefore never better for i to choose $y_i(s) < \xi_j(s)$ or $\lambda_j(s) < 0$ (a.e.). ☐

In the *incomplete* information case, each type $i \in \mathcal{I} = [0, 1]$ must choose its own acceptance time by optimization of (6) while the offer $y_i(s)$ is common to all types in $\mathcal{I}$. In order to construct a PBE we first need to define j's strategy. One can show easily that $\xi_i(s)$ is increasing in i so that $\xi_{i=\phi_i(s)}(s) = \min_{i \geq \phi_i(s)} \xi_i(s)$.[35] So, any $y_j(s) < \xi_{i=\phi_i(s)}(s)$ is "unacceptable" since all $i \geq \phi_i(s)$ that j believes he faces would prefer to fight to the end rather than accept it. Formally

Definition 2: j's offer y_j is "acceptable" (given beliefs ϕ_i) if $y_j(s) \geq \xi_{i=\phi_i(s)}(s)$. It is "positive" if the inequality is strict and "unacceptable" otherwise.

If purification is to succeed, the beliefs $\phi_j(t)$ will have to converges to the mixed strategy $\phi_j(t)$ of Definition 1 as $\epsilon \to 0$. In that definition λ_i and the corresponding ϕ_j resulted solely from j's strategy. So, $\lambda_i(s) < 0$ did not occur. But in the incomplete information case, $\lambda_i(s)$ appears with $i = \phi_i(s)$, the beliefs ϕ_i and ϕ_j become entangled and the case $\lambda_{i=\phi_i(s)}(s) < 0$ can result from i's choices. And since λ_i serves as a hazard rate for ϕ_j it must remain non negative. Moreover, beliefs $\phi_j(s)$ are strictly increasing whenever $\lambda_{i=\phi_i(s)}(s) > 0$. This means that j is being actively screened. We make

Definition 3: j's terms are "acceptable" if his offer is acceptable and it determines $\lambda_{i=\phi_i(s)}(s) \geq 0$ (given i's terms). j's terms are "positive" if his offer is positive and it determines $\lambda_{i=\phi_i(s)}(s) > 0$ (given i's terms).

Beliefs will be described by the (continuous) solution

$$\phi_j(t) = 1 - e^{-\int_0^t \lambda^*_{i=\phi_i(s)}(s)ds} \tag{A7}$$

of $$\frac{D_+\phi_j(s)}{1-\phi_j(s)} = \lambda^*_{i=\phi_i(s)}(s) = \begin{cases} \lambda_{i=\phi_i(s)}(s) & \text{if both sides' terms are acceptable} \\ 0 & \text{otherwise} \end{cases}$$

and symmetrically, by exchanging i and j.[36]

Definition 4 (Assessment): Assume beliefs according to (A7) and each type's strategy (here written for j) defined by:

(1) j offers terms that are always acceptable;

(2) j rejects i's terms when they are not acceptable;

(3) Otherwise j accepts $y_i(t_j)$ by time t_j that maximizes $\mathbb{E}\pi_j(t)$.

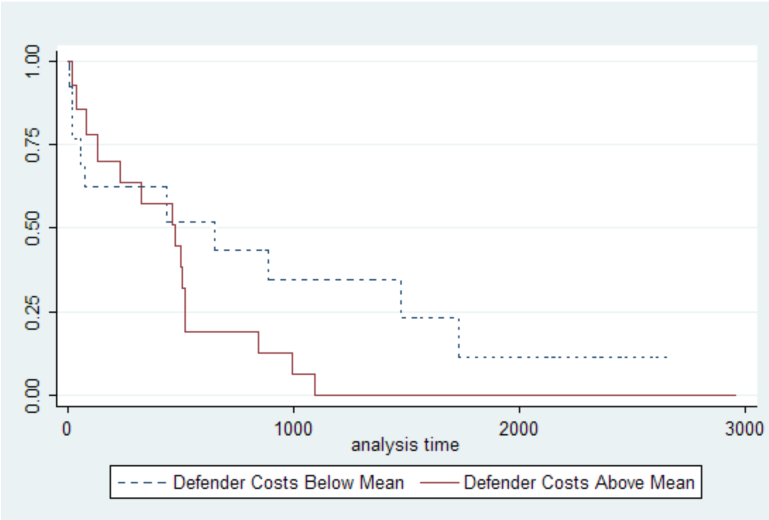
In order to obtain a PBE, one needs to verify that a symmetric strategy by i is sequentially rational in response and that the beliefs in (A7) are consistent with the two sides' strategies. One further needs to characterize the optimal offer functions y_i and verify that they are indeed continuous. Because of its highly technical nature we only state the result and make its proof available upon request.

Theorem 2: For any $\epsilon > 0$ there exists an assessment according to Definition 4 that forms a PBE. Moreover, such PBEs cannot involve positive terms. And as $\epsilon \to 0$, the PBEs converge to the SPE of Definition 1 provided each side offers the other its BATNA.

So, the SPE that forms the basis of our empirical estimates is indeed purifiable provided the two sides offer each other the minimum they can guarantee themselves by fighting to the end.

Graph 1

Challenger Kaplan-Meier Survival Estimates when Challenger Backs Down



Graph 2

Defender Kaplan-Meier Survival Estimates when Defender Acquiesces

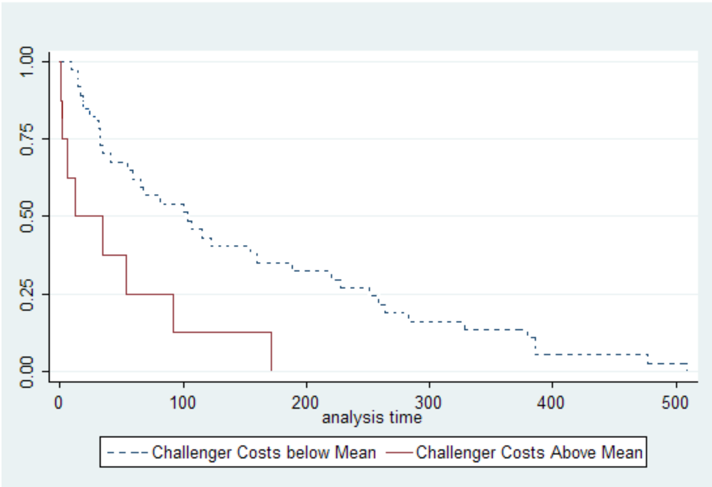


Table 1

	Challenger Gives In		**Defender Gives In**	
	Mean	Median	Mean	Median
War Duration (months)	22.01	14.91	3.70	21.97
cmilpopshare %	5.55	0.968	**3.99**	**0.880**
dmilpopshare %	**1.18**	**1.26**	1.69	0.716
crelativecap (c/d ratio)	**0.434**	**0.367**	0.720	0.807
drelativecap (d/c ratio)	0.566	0.633	**0.280**	**0.193**
csalience	**0.9**	**1**	0.812	1
dsalience	1.05	1	**1.189**	**1**

Table 2

Challenger Gives In: Hazard Estimates

	Model 1		***Model 2***		***Model 3***		***Model 4***	
Variables	Coefficient	p > $\lvert z \rvert$	Coefficient	p > $\lvert z \rvert$	Coefficient	p > $\lvert z \rvert$	Coefficient	p > $\lvert z \rvert$
dmilpopshare	0.955**	0.016	0.993**	0.018	0.991**	0.013	0.867**	0.010
cmilpopshare			-0.005	0.794				
dmilpop%change					0.002	0.47		
crelativecap	-11.544**	0.000	-11.830**	0.000	-11.446**	0.000	-9.276**	0.000
crelativeflowloss	1.527	0.386	1.480	0.400	1.408	0.435		
crelativequal	-0.262**	0.009	-0.257**	0.010	-0.275*	0.009	-0.208*	0.091
terrain	-5.843**	0.000	-5.970**	0.001	-5.661**	0.000	-4.964**	0.002
contiguity	0.305*	0.062	0.317*	0.062	0.320*	0.059		
csalient	-4.598**	0.002	-4.704**	0.002	-4.453**	0.003	-4.206**	0.001
caudience	0.440	0.127	0.456	0.125	0.421	0.188		
Constant	2.659	0.253	2.757	0.240	2.044	0.400		
p (duration parameter)	1.519**		1.548**		1.556**		1.322**	
Log-Likelihood	-20.27		-20.76		-20.56		-22.86	
Aikaike Info. Criterion	**60.54**		**63.52**		**63.12**		**59.72**	
Mean abs. error (months)	25.01		24.80		21.74		25.30	
Median abs. error (months)	6.89		7.09		3.69		5.20	
Mean error to war length	1.86		1.87		1.59		2.06	
Number of wars[1]	20		20		20		20	
Number of data points	55		55		55		55	
z-statistics are in parentheses. One tailed significance: *p<0.05 **p<0.01. *crelativequal* is not available for all wars in the data set								
[1]Estimation uses a subset of the data as data to compute *crelativequal* is missing for some wars.								

Table 3

Defender Gives In: Hazard Estimates

	Model 1		***Model 2***		***Model 3***		***Model 4***	
Variables	Coefficient	p > $\|z\|$	Coefficient	p > $\|z\|$	Coefficient	p > $\|z\|$	Coefficient	p > $\|z\|$
cmilpopshare	0.096**	0.001	0.092**	0.001	0.085**	0.009	0.091**	0.000
dmilpopshare			0.133	0.081				
cmilpopsharedpct					0.001	0.255		
drelativecap	-1.725**	0.017	-2.060**	0.007	-1.786**	0.014	-1.436**	0.029
drelativeflowloss	-0.549	0.610	0.019	0.987	-0.252	0.818		
drelativequal	0.013**	0.000	0.013**	0.000	0.013**	0.000	-0.011**	0.001
terrain	-3.210**	0.002	-3.374**	0.001	-3.207**	0.002	-2.519**	0.006
contiguity	-0.079	0.490	-0.076	0.510	-0.106	0.366		
dsalient	-0.398	0.234	-0.392	0.260	-0.371	0.272		
daudience	0.196*	0.070	0.194	0.073	0.219**	0.049		
Constant	-3.017**	0.016	-3.480**	0.010	-3.335**	0.011	-4.071**	0.001
p (duration parameter)	1.329**		1.354**		1.357**		1.250**	
Log-Likelihood	-46.80		-45.81		-46.21		-48.69	
Aikaike Info. Criterion	**113.60**		**113.62**		**114.42**		**109.38**	
Mean abs. error (months)	2.39		2.40		2.47		2.46	
Median abs. error (months)	1.25		1.37		1.17		1.57	
Mean error to war length	1.39		1.34		1.43		1.43	
Number of wars[1]	37		37		37		37	
Number of data points	48		48		48		48	
z-statistics are in parentheses. One tailed significance: *p<0.05 **p<0.01								
[1]Estimation uses a subset of the data as data to compute *drelativequal* is missing for some wars.								

Table 4

Comparative Goodness of Fit

The Challenger Gives In			
Model		**Mean**	**Median**
Model 1	*Observed*	22.06	14.91
	Predicted	33.58	7.15
	Absolute Error	25.01	6.89
	Abs. Err/Observed	1.86	0.35
Slantchev	*Observed*	22.06	14.91
	Predicted	64.68	31.30
	Absolute Error	45.56	17.49
	Abs. Err/observed	8.04	2.57
The defender gives In			
Model 1	*Observed*	3.70	1.97
	Predicted	3.84	3.44
	Absolute Error	2.39	1.25
	Abs. Err/Observed	1.39	0.59
Slantchev	*Observed*	3.70	1.97
	Predicted	5.42	3.13
	Absolute Error	3.93	1.75
	Abs. Err/observed	2.58	0.83

Table 5

The Importance of Attrition Behavior:

Change of Independent Variable from Median Holding all Others at their Medians

CHALLENGER GIVES IN						
	Variable Values			**Variable Impact in Months**		
				From Median		Span
Variable	***Median***	***1stQ***	***3rdQ***	***To 1stQ***	***To 3rdQ***	***1stQ to 3rdQ***
dmilpopshare %	1.259	0.279	1.795	8.77	-2.94	11.71
crelativecap	0.367	0.180	0.674	-7.79	96.17	103.96
terrain	0.8	0.6	0.9	-5.51	4.83	10.33
DEFENDER GIVES IN						
Variable	***Median***	***1stQ***	***3rdQ***	***To 1stQ***	***To 3rdQ***	***1stQ to 3rdQ***
cmilpopshare %	0.881	0.498	2.466	0.75	-0.31	1.06
drelativecap	0.193	0.052	0.465	-0.48	1.18	1.66
terrain	0.6	0.5	0.8	-0.61	1.74	2.35

Footnotes

[1]See for example Slantchev 2003a, Langlois and Langlois 2006. Slantchev appeals to "extremal equilibria" to explain war between informed states who can bargain. In these equilibria, the adversaries *agree to fight* for some time before settling on a division of the pie under the threat of reversion to the worst equilibrium of the deviator. langlois and Langlois explain fighting between informed rivals as a deterrence strategy when one state holds the prize at the outset and another challenges for it. The defender's threatens escalation of the dispute to war with some likelihood in case of challenge and the challenger backs up the credibility of his challenge by being willing to fight with probability if it comes to that. As a result, fighting will occur with some likelihood.

[2]See for example Filson and Werner 2002, Slantchev 2003b, and Powell 2004. Filson and Werner provide a particularly crisp account of the process by which an attacker will calibrate his demands according to his beliefs about the defender's strength and the outcome of battle. In their model, fighting uses finite resources and this modeling choice distinguishes their model from Slantchev's (Slantchev 2003). In contrast to Filson and Werner, Slantchev allows for a process of offers and counteroffers, so that the informed party has an opportunity to strategically take advantage of his rival's uncertainty in the setting of his own demands. While these authors emphasize the impact of negotiation outcomes as well as battle outcomes in the process of convergence, they develop their models assuming a small number of types. Slantchev distinguishes between three types and Filson and Werner assume only two. In Powell's war and bargaining model (Powell 2004), the uninformed party is first described as screening the continuum of types according to a schedule that corresponds to the frequency of decision periods. Powell examines the impact of allowing for multiple offers within the interval of time that separates battles and finds that uncertainty about costs would lead to immediate

acceptance of an offer while uncertainty about the chances of winning a battle will take at least one battle to resolve. While these models differ in a variety of ways, they all adopt the classic screening process that gradually increases settlement offers as the strength of the enemy is revealed. While low offers will be rejected by strong types in these models, all offers are serious in the following sense: the offers are tailored to a type and would be acceptable to that type.

[3]O'Ballance 1964, 245-46 as quoted in Pillar 1986, 79.

[4]Webster defines attrition as the "act of weakening or exhausting by constant harassment, abuse, or attack." See http://aolsvc.merriam-webster.aol.com/dictionary/attrition.

[5]Smith and Stam state that they include war of attrition aspects in their model (Smith and Stam 2004). But the driver of war duration is the convergence of beliefs, it is not one side's decision to give in to the other.

[6]Smith develops a war of attrition model to describe sanctioning behavior but does not allow for bargaining between states during the sanctioning episode (Smith 1996).

[7]For example, Japan and China engaged about the same proportion of their populations in the military during the war of 1894, but COW data shows that Chinese forces were ten times larger than Japanese forces.

[8]The data we use is a subset of the data used by Slantchev (2004). His data set includes wars that are disgregated from multilateral wars and wars that are excluded from COW because of "restrictive requirements for system membership" (Slantchev 2004, 818) following Dupuy and Dupu 1985, and Bennett and Stam 1996.

[9]We could instead introduce an arbitrary lower bound or ratio to j's forces below which i's forces are defeated.

[10]Cases where $q_i = 0$ or $q_j = 0$ are not substantially different.

[11]There are orbits that evolve to a stalemate. But they have nil probability.

[12]Note that, by contrast, the sign of $\frac{\partial c_i(z_i,z_j)}{\partial z_j}$ is ambiguous as the relationship between z_i and z_j changes with the war dynamic.

[13]Technically, we assume that $u_i(0) = 0$, that $u_i' > 0$ is bounded above and below by positive numbers, and that $u_i'' \leq 0$ (weak concavity).

[14]If $t \geq \theta$, $\pi_i^t \equiv u_i(x_i^*)$ and $\sigma_i(t)$ does not just involve sunk costs since there are none between t and θ.

[15]When one side makes an offer that matches the other side's standing offer, meaning $y_i + y_j = 1$, it is deemed instantly accepted and the game ends with the players' permanent and peaceful enjoyment of their respective shares.

[16]A left-discontinuity in y_j at s would result in $D_+ u_i(y_j(s))$ being a Dirac mass and ϕ_j being discontinuous at s. However, all technical developments can be done with continuous offer functions y_j, analyzing discontinuous offers as the limit of continuous ones since their effect only appears in the integral (6). Moreover, purification only yields the continuous $y_j \equiv \xi_{\mathcal{I}}$ in equilibrium.

[17]Of course, Britain's bargaining stance at Ghent was also colored by the ongoing European conflict (Pillar 1983, 204.)

[18]It is also important to point out that the theory links i's flow costs to j's hazard rate and not some ratio of i and j's costs to j's hazard rate. The relationship is about j's resilience in the face of i's war effort whatever j's own war effort might be. In fact, j's war effort is itself a reflection of i's resilience regardless of i's choice of effort.

[19]Our proxies for challenger and defender war costs are labeled *cmilpopshare* and *dmilpopshare* respectively. They are only weakly correlated to each other or to measures of challenger or defender strength such as them COW capability index or the size of its military force. The correlation coefficients are as follows:

Defender Gives Up						
	cmilpopsh	dmilpopsh	ccapab	dmilitper	csizemil	dsizemil
cmilpopshare	1.0000					
dmilpopshare	-0.0551	1.0000				
cCOWcapability	0.2779	-0.1291	1.0000			
dCOWcapability	-0.1610	-0.0095	-0.0309	1.000		
csizeof military	0.0468	0.1152	0.3482	-0.1015	1.000	
dsizeofmilitary	-0.1417	0.0783	-0.0289	0.7721	0.2473	1.000
Challenger Gives Up						
	dmilpopsh	cmilpopsh	ccapab	ccapab	dsizemil	csizemil
dmilpopshare	1.0000					
cmilpopshare	0.1543	1.0000				
dCOWcapability	0.0272	0.1953	1.0000			
cCOWcapability	-0.0160	0.0509	0.4376	1.0000		
dsizeofmilitary	0.1338	0.7001	0.5773	0.1456	1.0000	
csizeofmilitary	0.1601	0.8372	0.3623	0.2684	0.8569	1.0000

The first column of the above table records correlation between state i's war costs, j's war costs and measures of i and j's military capacity for the two sets of wars that we examine. Correlations are typically low. It is also noteworthy that *cmilpopshare* and *dmilpopshare* are only weakly correlated to issue salience as measured by Slantchev (2003) and labeled *csalient* and *dsalient* respectively:

Challenger Gives in

	cmilpopshare	*csalient*
cmilpopshare	1.000	
csalient	0.0018	1.000

	cmilpopshare	*csalient*
dmilpopshare	1.000	
dsalient	− 0.1868	1.000

Defender Backs Down

	cmilpopshare	*csalient*
dmilpopshare	1.000	
dsalient	− 0.1328	1.000

	cmilpopshare	*csalient*
cmilpopshare	1.000	
csalient	0.1231	1.000

These data indicate that strategic variables *cmilpopshare* and *dmilpopshare* are not proxies for each state's perception of the salience of the issue they are fighting over.

[20]One might argue that battle deaths as a proportion of the population could also measure a state's willingness to incur costs. However, by bringing in the outcome of the actual fight, this measure does not capture the strategic intent that we seek to reveal by linking the extent of a state's war mobilization to the rival's expected backdown. The proportion of the population in the military captures the opportunity cost of war that is traded off

against the benefit of the rival's backdown. Battle deaths per capita are the *result* of the strategic decision to go to war not a strategic element in the decision to fight.

[21]Information on the location of the battlefield for the wars in our database is, surprisingly, hard to find. Singer and Small 1972, provide that information for the wars they examine. Using this source we found that for the 33 wars that are in both our data bases, the battlefield was identified as the defender's territory for 78% of the wars.

[22]Salience is defined as follows: Following Slantchev 2004, 819, issues of "regime/state survival, national liberation or autonomy" are of critical salience; issues of "territory, integrity of state or honor/ideology" are of medium salience; and issues of "empire maintenance, commercial disputes or policy" are assigned minimum salience.

[23]Assuming a log logistic, or a Gompertz distribution for the unknown base hazard did not improve the hazard estimations. Assuming a Weibull distribution minimized the Aikaike Information criterion.

[24]An examination of the change *dmilpopshare* reveals that there is frequently a surge at the end of a long war while *dmilpopshare* remains steady or even decreases in some of the intervening war years. For example, in the First Anglo-Burmese war of 1823, Britain increased the proportion of its population in the military by 11% in the last year of the war (1826) but decreased it by 1.4% in 1825. Thus, while the level of the variable *dmilpopshare* significantly affects war duration in line with our attrition hypothesis, its change over time is not a good predictor of war duration as the data suggest that it is not strategically managed in most war years: a final surge may precipitate the end of a war, but it may not affect the intervening duration of the war.

[25]These variables measure different attributes of the challenger's military might: the correlation of -0.1461 between *crelativecap* and *crelativequal* is low and negative.

[26]Slantchev's model uses the following explanatory variables:*military parity* $=$ absolute value of the difference between the party's military forces scaled to fall between 0 and 1, *reserveparity* which is measured in the same way but considers the difference in populations between the protagonists, *terrain*, *contiguity*, *total population* $=$ sum of challenger and defender populations, *total military* $=$ sum of challenger and defender military forces, *Initiator democracy dummy* $=0$ if the initiator's democracy score is less than 6, 1 otherwise. As all the wars we look at involve 2 states we do not need to control for number of states.

[27]It is also the case that our model offers a better fit to the data than Slantchev's if prediction improvement is measured against a naive model that fits a constant only model to the data (see Bennett and Stam 2008). We improve the mean absolute error to war length relative to a naive model by 85.2% when the challenger gives in and 79.9% when the defender gives in. By contrast, Slantchev's model on these data sets improves on the naive model by 36.3% and 62.7% respectively. By comparison, according to this measure, Bennett and Stam improve on the naive model by 67.3% when treating observation per war year and 69.2% when using one observation per war.

[28]For completeness, although the case has probability zero, we assume that *simultaneous* overgenerous offers $y_i + y_j > 1$ are deemed null.

[29]An increasing λ_i would require swapping the types so that ϕ_j would be decreasing.

[30]To prove Lemma 1 write $\mathbb{E}\pi_i(t+s)$ as in (6) and replace according to (4) and (A2).

[31]This does not mean that discontinuous PBEs cannot exist. It only means that the optimal response to continuity is continuous.

[32](a.e.) means "almost everywhere". It could fail without consequences on a subset of time that would not count in the integral (i.e., would have measure zero).

[33]The condition $\lambda_j(t) < 0$ holds when $D_+ y_i(t)$ is positive with a high enough magnitude. Intuitively, it is too soon for j to accept since i's offer is headed for much higher levels.

[34]At $t = 0$ the result holds trivially. Subtracting (A5) from (A6) and differentiating with respect to t yields the needed inequality: $\big(\pi_j(\omega_i^t) - \pi_j(\omega_j^t)\big) D_+\phi_i(t) \geq \phi_i(t) D_+\pi_j(\omega_j^t)$ which always holds if $D_+\pi_j(\omega_j^t) \leq 0$ and is ensured by $\lambda_j(t) \geq 0$.

[35]A proof is available upon request.

[36]The beliefs must be defined even in case a deviation results in $\lambda_i < 0$. A discontinuity in y_j at s would result in a Dirac mass in λ_i^* and a corresponding Dirac mass in $D_+\phi_j$ according to (A7) and ϕ_j would be discontinuous at s.